

Comments to topological defects in bilayer vesicles

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To explain the details of bilayer vesicle aggregation, we revised the anyon model for lipid domains formation in closed vesicles of lipid-cholesterol system *DPPC/DLPC*/cholesterol, which was measured by Feigenson and Tokumasu (Biophys. Journal, 2001, 2003) in frames of the different optical experiments and atomic-force microscopy.

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Introduction

The idea by Park with coworkers [1], and by Evans [2] about the orientational order parameter in closed liquid crystal aggregates of spherical and non-spherical shapes, has been almost illustrated on the Feigenson's experiments [3]. The nanoscopic domains formation of the giant unilamellar vesicles (GUV's) surface, consisting of dipalmitoylphosphatidylcholine (DPPC)/dilauroylphosphatidylcholine (DLPC)/ cholesterol mixed bilayers, were explained by the fractional quantum Hall effect analogy [4].

However, the confocal fluorescent microscopy (CFM) and the fluorescence resonant energy transfer (FRET) measurements [3] are not capable to identify the real sizes of these surface structures, predicted for the "D" - region of the phase diagram [3]. Using the atomic force microscopy (AFM) on the large unilamellar vesicles (LUV's) [5], the regions of nanoscopic domains, as well as of microscopic domains, became observable in "A", "B", and "C" - regions in addition, where the inter-layer bulk lipidic clusters were found. Applied LUV suspension on freshly cleaved mica [5] in AFM measurements render a native GUV's environment of the same lipids closely resembling.

The data of [5] revealed the significant features of the domains formation. If inside of the "A", "B", and "C" bilayer regions, the lipidic clusters exist [5], the model with anyons, in which FQHE formalism has been used [4], may be corrected.

Now looking to [5], disordered closed vesicle axes stabilization is caused not only by spontaneous appearance of the surface lipid domains, but also by internal bilayer forces, forming the bulk defects (hedgehogs). Hedgehogs prevent us to use the bilayer thickness as a parameter, which is connected the manifold of two spheres $M_1 = S_1^2$ and $M_2 = S_2^2$, corresponding to an outer- and an inner monolayer of a closed vesicle, respectively.

We suspect the composition of all observed defects can be described in frame of non-Abelian statistics (that could be applied only for surface vortexes alone). These common topological properties are observable in superfluid *He* phases, liquid crystals and superconductors [2, 6, 7, 8].

Since the geometry of the inner GUVs sphere is not viewed by means of CFM et. c. [3], and only the nanoscopic clusters of about 46 nm diameters are dispersed throughout of all areas in LUVs, we can discuss the different mechanisms of fractional axis shape stabilization, associated with defects of both types.

1. Formalism

Using mean-field approach for lowest Landau levels with some gauge parametrization, Evans found [2] the partition function of the vesicle

$$Z = 2\pi \int e^{-F_{eff}(t, \mathbf{n}_1, \mathbf{n}_2)} t^5 (1 - \mathbf{n}_1 \mathbf{n}_2) dt d\Omega_1 d\Omega_2 \delta(\mathbf{n}_1 \mathbf{n}_2 - \chi) d\chi, \quad (1)$$

where $F_{eff} = F_{eff}(\chi, t)$ is the effective potential, $-1 \leq \chi \leq 1$ is a measure of the relative separation of two vortexes, $d\Omega_k = \sin\theta_k d\theta_k d\varphi_k$, with the orientational variables in spherical coordinates θ_k, φ_k . 'Modified normal' unit vectors from the center of vesicle toward the k^{th} vortex obey $\mathbf{n}_1 \mathbf{n}_2 = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2)$, t is a measure of the overall real amplitude of the order parameter.

(The basic parametrization is given by the measure

$$D[\rho] = \prod_{\Omega} \hat{\mathbf{r}} \cdot \mathbf{N} d\rho(\sigma), \quad (2)$$

where Ω are all solid angles, $\rho(\sigma)$ is the scalar field of two-dimensional surface coordinates $\sigma = (\sigma_1, \sigma_2)$, and $\hat{\mathbf{r}} \cdot \mathbf{N} = 1 - \frac{1}{2}\{(\partial_{\theta}\rho)^2 + \csc^2\theta(\partial_{\varphi}\rho)^2\}$.)

All this means the surface vortexes generate the set of six thermodynamical parameters, which can be presented in three more or less physically combinations (of 'temperature', 'malleability', and the scalar parameter) [2]. And therefore the filling factor ν [9] takes the easy equatable values in the anyons terms.

2. Several ways of vesicle shape evolution under hedgehogs

In bulk *Sm-C* aggregates, hedgehogs having the group $\pi_2(S_2/Z_2)$, may be of hyperbolic and spherical structures and be described in torus degeneracy space [10, 11]. They always can be taken up (let out) by the vesicle surface vortexes [12, 13].

So with the advent of the hedgehogs, at certain thermodynamical conditions (temperature and/or lyotropic content of a vesicle), it is possible to find the next positions of defects, and of the anyon states.

2.1. Coexisting of hedgehogs with surface vortices

Here all hedgehogs play role in an external potential for Laughlin wave function [9].

2.2. Destruction of hedgehogs

In case of raising of the hedgehogs to the (for instance, to the outer S_1^2) surface, the effect of influence $\pi_1(R)$ group onto $\pi_2(R)$ can be observed [8], since $\pi_1(R)$ group is non-trivial. According to [14], the field lines between hedgehogs can collapsed to 'string', connecting a hedgehog with a surface vortex.

Comments

At continuous phase transition in the vesicle, caused by variation of aggregate content, the broken symmetry can be quite observable (at $\pi_2 \rightarrow \pi_1$ homomorphism), in agree with marginal type-I-type-II vesicle [2].

As fractionalization associate with the groundstate degeneracy [10], the scenario in the hedgehogs presence is noncontradictory against a background of anyons.

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